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N62-15385

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SOCIETY OF AUTOMOTIVE ENGINEERS

National Aeronautic Meeting
April 3-6, 1962
New York, New York

513E

p 207

Paper to be presented at SAE National Aeronautic Meeting,
April 3-6, 1962, New York

MIDCOURSE NAVIGATION FOR A MANNED LUNAR MISSION

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INTRODUCTION

A study of midcourse guidance for manned lunar missions at Ames Research Center has resulted in the concept for a feasible guidance system to be described in this paper. First our ideas of the basic system requirements which serve as guidelines in the design are outlined. The design approach is then presented and the scheme of the resulting system is described. Finally, the results of a digital computer simulation are given which indicate that such a system could provide excellent guidance for a manned lunar mission. A discussion of the mathematics involved is omitted here to save time; a more complete treatment is given in references 1 and 2.

To set the midcourse guidance problem in proper context, consider figure 1 which illustrates a figure eight circumlunar trajectory. Such a trajectory is perhaps typical of what might be accomplished in an early manned lunar flight. Phase I includes boost from the launch pad and terminates with injection into the outgoing trajectory. Phase II, beginning at injection, is termed midcourse. During this phase the vehicle is essentially in free fall except for short periods of accelerated flight when velocity corrections are made. Such corrections

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are required because of the extreme sensitivity of the trajectory to small injection errors; for example, a 1-foot-per-second velocity error can result in a several thousand mile miss upon return to Earth. The midcourse phase is considered complete upon entry into the Earth's atmosphere and terminal guidance (phase III) proceeds from this point to landing.

SYSTEM REQUIREMENTS

Midcourse guidance is seen to be a control problem like many others, albeit more complex than many. In broad outline, the problem may be represented in the form shown in figure 2. The first box represents the data acquired by both an Earth-based tracking network and on-board sensors of unspecified type. These data are used in the next box to compute a best estimate of the vehicle's position and velocity (which we call the state). From this estimate a prediction is made of the end-point conditions (e.g., the perilune or perigee of the estimated trajectory). Then a guidance law is employed to compute the velocity correction required to change the estimated end-point conditions to correspond to those desired, and a decision must be made as to whether this velocity correction should be applied now or action delayed until a later time. Finally, if the decision logic so indicates the velocity correction is implemented. Since the first velocity correction is not perfect, the acquisition of data continues throughout the flight, and several corrections may be made.

Of course, we must be more specific about the control system configuration. In order to do this, we must first consider what requirements other than the normal function of guidance to an end point the

system must meet. Notably, the fact that a manned mission is involved has a pronounced effect on the system design. This is because, first, we wish to make the most of man's inherent reliability and his unique ability to perform complex tasks, and second, we must realize the ultimate in reliability and system versatility to ensure the safe return of the crew. These considerations lead us to the conclusion that besides the obvious ground-based tracking and computation, a complete and versatile on-board capability must be provided. This on-board system would include tracking, computation, display, and control features under the direct command of the crew. Maximum performance would be obtained by providing the on-board system with information obtained via communication link from the Earth, but the on-board system must be capable of at least safe return performance without such information. In this paper we will concentrate on the design of this critical on-board system, with only minor references to the implied Earth-based tracking and computations and the Earth-vehicle communication link.

DESCRIPTION OF THE ON-BOARD SYSTEM

From the previous argument it is seen that the on-board system must include all the features illustrated in figure 2. In more detail, this system can be represented in block diagram form as shown in figure 3. To the left of the dotted line is a representation of the kinematics. Injection conditions, acting through the trajectory dynamics, produce the state, or position and velocity of the vehicle. The space angles which can be observed from the vehicle are related

to the state by geometry equations. The actual measurements obtained by means of the optical instruments differ from the true angles because of observation errors, which are represented as inputs to the system in the figure.

To the right of the dotted line the control system is shown. Note that the crew is in command of all guidance functions. First optical tracking measurements are made by manual operation of the instruments. The data so obtained are evaluated and then inserted, perhaps by keyboard, into the digital computer. These data are also transmitted via a communication link to the Earth where they are used along with Earth-based tracking data to compute the vehicle's trajectory. Periodically, under normal circumstances, the results of the Earth computations are transmitted back to the vehicle for evaluation and comparison with the on-board calculations.

The decision to execute a velocity correction is made by the crew, with the aid of information supplied by the computer. Upon command the computer calculates the magnitude and direction of the correction to be made. The vehicle is then properly oriented and the velocity correction is initiated. The correction is monitored by means of accelerometers. The measured correction, which, in general, will differ from that commanded because of errors in the control mechanism, is inserted into the computer and transmitted to the Earth for inclusion in the trajectory calculations.

The control system loop is closed by way of the influence which the velocity correction has upon the state, acting through the trajectory dynamics. The over-all system inputs are seen to be the injection conditions and the observation errors.

Now that we have identified the principal parts of the control system and their functions, we wish to consider in more detail the computations that must be carried out in the on-board digital computer. The trajectory estimation portion of the computations is in essence a filtering problem, and our design uses a specialization of a general theory of multivariable linear filtering proposed by R. E. Kalman (ref. 3). The input to the filter is a sequence of angle measurements, and the output is the best estimate of the trajectory which can be obtained from these data. Besides the optimal estimate, the computations prescribed by the theory include the statistics of the errors in estimation - in other words, how good is the current estimate. The computations involved in trajectory estimation are shown in block diagram form in figure 4. The operation of the system is best understood if the following sequence of events for one observation is considered:

First, an observation is made of either the Earth or the Moon. This observation consists of measurement of the angles between a point on the body being observed and certain fixed stars and perhaps the subtended angle of the Earth or Moon. The angles and the time of the observation are recorded. These raw data then are directed to both the Earth computation center and the on-board computer.

Second, the computer integrates the equations of motion from the time of the last observation until computer time equals observation time. Using the geometry equations, the computer then calculates what the observed angles should be, on the basis of the updated best estimate of the trajectory. The differences between the observed and computed angles are calculated, along with the expected rms values of these differences, which can be used as a check to determine whether or not the data are reasonable. If this check fails, it is likely that a mistake in the observation has been made and corrective measures are taken, repeating the observation and checking with the Earth computation center.

Third, the verified data are used. The manner in which the computer operates upon the data is to:

(1) Compute an optimum weighting function, K, for the particular type measurement made;

(2) Multiply the difference angles by the weighting function and modify the estimate of the trajectory at the observation time by the result.

For the best use of all available data under normal circumstances, the results of ground computations of the trajectory would be obtained from the Earth computation center at periodic intervals, perhaps every 6 to 12 hours. This information would be compared with on-board data by the crew and after sufficient cross-checking would be inserted into the computer in place of the results of the on-board computations. It is envisioned tentatively that this would be accomplished via a manual computer keyboard rather than telemetering and automatic read-in. The on-board estimate of the spacecraft's position and velocity is,

therefore, as good as the ground estimate at these times. Since the ground estimate is computed from both Earth-based tracking and on-board data, this represents the best possible estimate.

The next subject for consideration is the midcourse velocity correction calculations. Figure 5 shows the scheme we have conceived for this purpose. Essentially the problem is, given the estimated trajectory, that is, the position and velocity vectors, to predict the expected "miss" at the end point and compute a velocity correction which would make this miss zero. For our study we have used a fixed time of arrival navigation concept. In essence, we guide the vehicle on the outgoing leg to arrive at a particular perilune at a specified time, and on the incoming leg to enter the atmosphere at a particular point at a specified time. The fixed time of arrival feature is not absolutely essential and further study is under way to determine the advantages of a more flexible scheme.

If the vehicle equations of motion are linearized around a reference trajectory which passes through the desired end point, then deviations from the end point - that is, the miss - can be expressed as a linear function of the deviation in position and velocity from the reference trajectory at the present time. This relationship is the prediction matrix. By computing the difference between the reference and estimated state vectors and multiplying by the prediction matrix we obtain the predicted miss. Then by a further application of a portion of the prediction matrix which constitutes our guidance law we compute the required velocity increment. The prediction matrix

also enables us to compute the statistics of the prediction error, which is useful in indicating how well we will do in our guidance task.

When the crew decides, probably on the basis of a predetermined schedule and after checking with the Earth computation center, that the computed velocity correction is to be applied, the correction is executed, measured by means of integrating accelerometers on an inertial platform, and the measured correction is introduced into the trajectory calculations in the same manner as an observation. Thus, the velocity correction in no way interferes with the step-by-step trajectory estimation procedure. If an abort is to be executed, the velocity correction is a large one compared to simple guidance corrections, and, of course, different prediction and guidance equations are employed since the desired end point is different. Nevertheless, the method of operation, in general, is the same.

The application of linear prediction methods has some problems if large launch time variations are to be accommodated. In this case a single reference trajectory is not satisfactory because large deviations from the reference introduce nonlinear effects that produce sizable errors in the prediction. To have a large launch window, this system requires a number of on-board stored reference trajectories. The crew would then choose from this list the reference trajectory which most closely fits the measured injection conditions, or the selection might be made automatically within the computer.

The linear prediction concept when used in the case of an abort also requires stored reference trajectories corresponding to a large number of different abort locations. Further study is under way to determine the number of references required or some means of circumventing this problem.

With the trajectory determination and velocity correction techniques outline, it still is necessary to specify an optimum schedule for the on-board observations and velocity corrections, that is, how many and at what times should observations and corrections be made. The choice of such a schedule should take into account the expense of the on-board operations, in particular, the fuel expended for reorienting the vehicle if necessary, and the power expended in operating the instruments and the on-board computer. By simulating the system on a digital computer, it is possible to find a schedule that allows mission objectives to be achieved at minimum expense. Fortunately, mission success is not particularly sensitive to variations in this schedule, so that the schedule need not be adhered to very closely during the actual flight.

RESULTS OF SIMULATION STUDY

In order to evaluate the performance of the midcourse navigation system described, a relatively complex simulation was employed. Before presenting some results, it is necessary to note the objectives and to understand the assumptions under which the simulation was conducted. One objective was to show that the mission could be accomplished by the

use of on-board optical measurements alone. As will be shown, this objective was achieved. Of course, the results are not as good as they would be if Earth-based tracking data were included. In other words, we are showing that mission objectives could be achieved with only part of the complete system in use, namely the on-board portion.

Shown in figure 6 is one type of schedule employed in the study of a 6-1/2-day circumlunar mission. An on-board optical device was assumed to measure the subtended angle and the right ascension and declination of the center line of either the Earth or the Moon at the points indicated on the trajectory. Tick marks are Earth observations and crosses, Moon observations. Note that this schedule for the most part alternates between Earth and Moon observations. No particular attempt has been made to optimize this schedule. The total number of observations is 77 for the entire 6-1/2-day flight.

The velocity corrections are shown on the trajectory as circles, the total number being 6. The first correction is made 13 hours after injection, followed by two others on the outbound leg and three on the return leg. At least 1-hour's delay is allowed between observations and velocity corrections to give time for vehicle reorientation and other procedures associated with the application of a correction.

The errors assumed in the study were as follows: First, the standard deviations of the errors in each of the three simultaneously observed angles were taken to be

$$\sigma = \sqrt{100 + (0.001\gamma)^2} \qquad \text{seconds of arc}$$

where γ is the half subtended angle of the body observed. For large distances from the observed body, γ is small and σ, the rms angle error, approaches 10 seconds of arc. For 200 km altitude above the Earth, σ for Earth observations is about 270 seconds. This error formula does not necessarily correspond to any actual optical instrumentation system under consideration but at least is felt to be fairly representative and, hence, suitable for our purposes.

Second, the errors at injection were assumed to have standard deviations of 1 kilometer and 1 meter per second in each of three geocentric Cartesian inertial coordinate directions. Third, the statistics of the errors in making the necessary velocity corrections were assumed to be represented by an rms error in direction of one degree, and an rms magnitude error of 0.1 meter per second. Finally, the rms error in measuring the velocity corrections was taken as 0.01 meter per second in each of the three coordinate directions.

The final measure of any guidance scheme for a manned space vehicle is, of course, its ability to position the vehicle in space so that safe entry can be made into the Earth's atmosphere. Presuming that we have a reference trajectory which provides this safe entry, then a measure of the guidance effectiveness is the difference between the actual and reference trajectories. Figure 7 illustrates the manner in which we represent this difference. The dotted line indicates the reference trajectory. For any point in time on the reference trajectory we can compute the probability that the actual trajectory will be within a given range, r, and a given velocity, v, from the reference. Similarly, with respect to the actual trajectory we can compute the

probability that the estimated trajectory will lie within a given range $\tilde{r}$ and velocity $\tilde{v}$ from the actual trajectory. The rms values of these deviations have been calculated for the entire simulated flight and will be presented here for the time of virtual perigee on the reference trajectory. It is well known also that a successful entry can be made if the virtual perigee altitude is within a given band. Therefore, the rms value of the variation in virtual perigee has also been computed.

Table I summarizes the rms perigee data obtained from the simulation studies. These results are the same as would be obtained by averaging, in an rms sense, the results of many flights having the same error statistics and schedule of observations and velocity corrections. In the first column results are shown for the type of flight we have described previously as the standard case. Note that the rms variation in virtual perigee is only 1.1 km, indicating a high probability of safe entry for the spacecraft. The next two numbers of 26.4 km and 23.8 m/sec for the rms position and velocity deviations from reference are given at virtual perigee but are of the same order of magnitude at the time of atmospheric entry. Since the spacecraft is entering at near escape velocity (11,000 m/sec), the rms velocity deviation of 23.8 m/sec indicates that the rms entry flight path angle variation is less than 0.002 radian, and the terminal guidance system can therefore easily control the range error. The next two figures are the rms values of the errors in knowledge of the position and velocity, 15.0 km and 13.2 m/sec. These figures are to the terminal

guidance system what the uncertainty in knowledge of injection conditions are to the midcourse guidance system. They result in an uncertainty in the landing location which we have not calculated. Of course, any tracking information acquired during the terminal phase would reduce this uncertainty.

The total corrective velocity required for making the 6 corrections for the 6-1/2-day flight for the standard case is about 20 m/sec. This figure can be used to establish the amount of on-board fuel required for a mission with a high probability of success.

The effects of two parametric changes from the standard case are shown in the next two columns of the table. The second column of the table shows the results obtained when a much larger number of observations were made. Increasing the number by a factor of 11 decreases all the terminal errors by a factor of 2 but does not substantially decrease the total fuel required. The greater accuracy attainable by more observations can be used to compensate for poorer observation accuracy as shown in the last column. In this case 844 observations were made but with errors five times greater than in the previous case. The perigee errors are seen to be comparable to those of the standard case which has one-eleventh as many observations.

The effects of two other parametric variations are not shown in the table. First, if it is assumed that the velocity corrections can be made and measured perfectly, it is found that there is no significant reduction in either perigee errors or total corrective velocity. Second, if the injection errors are increased by a factor of 5, there is no increase in the terminal error, but the total velocity increment

required increases by approximately the same factor of 5, indicating that the injection errors are quite effectively washed out by the midcourse guidance system and that the midcourse corrections are mostly a function of the injection errors.

It should be pointed out that in each case cited in the table the spacecraft should return well within the entry corridor required for a vehicle with $L/D = 1/2$.

Table II shows data for the time of reference perilune. The trends are similar to those indicated in table I, and it is left to the reader to make the obvious comparisons.

The total rms correction increments shown in tables I and II are quite dependent upon the rms injection errors assumed. Thus, before much confidence can be attached to the actual numbers quoted it is necessary to verify that the statistics employed are realistic. Also, it must be noted that the error statistics given must be somewhat optimistic because not all sources of system error have been included in the analysis. Further study is under way to settle such points.

CONCLUSIONS

The simulation results may be summarized in the following two statements: (1) The guidance accuracy attainable with the system studied is well within the requirements for return to an established entry corridor; and (2) the corrective velocity associated with achieving this performance is relatively small.

The principal advantage of the proposed system is its versatility, achieved without undue complexity. Specifically, the system gives the optimal estimate of the spacecraft's position and velocity at all times with an on-board computer which it is believed will be of reasonable size. Thus, guidance and navigation can be successfully accomplished if need be without reliance on tracking information transmitted from the Earth. The implication here is that an integrated ground-control on-board system would provide an excellent combination of redundancy for minimization of possible errors and also a versatile emergency operating capability. Thus a higher probability of mission success and crew safety could be assured.

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2. McLean, John D., Schmidt, Stanley F., and McGee, Leonard A.: Optimal Filtering and Linear Prediction Applied to a Midcourse Navigation System for the Circumlunar Mission. NASA TN D-1208, 1962.

3. Kalman, R. E.: New Methods and Results in Linear Prediction and Filtering Theory. Technical Report 61-1, Research Institute of Advanced Studies, Baltimore, Maryland, November 1960.

RESULTS AT PERIGEE-RMS VALUES

		STANDARD SCHEDULE	DENSE SCHEDULE	OBSERVATION ERROR × 5
	NUMBER OF OBSERVATIONS	77	844	844
MISS	PERIGEE VARIATION	1.1	0.6	1.2
MISS	r(km)	26.4	11.5	33.5
MISS	v(m/sec)	23.8	11.0	29.9
UNCERTAINTY	$\tilde{r}$(km)	15.0	7.7	26.9
UNCERTAINTY	$\tilde{v}$(m/sec)	13.2	6.7	23.5
	TOTAL APPLIED ΔV m/sec	20.0	15.3	22.3

TABLE I

RESULTS AT PERILUNE-RMS VALUES

		STANDARD SCHEDULE	DENSE SCHEDULE	OBSERVATION ERROR × 5
	NUMBER OF OBSERVATIONS	77	844	844
MISS	PERILUNE VARIATION	2.3	1.7	3.5
MISS	r(km)	8.6	4.8	13.5
MISS	v(m/sec)	2.2	1.2	3.0
UNCERTAINTY	$\tilde{r}$(km)	2.9	0.8	3.5
UNCERTAINTY	$\tilde{v}$(m/sec)	0.27	0.08	0.32
	TOTAL APPLIED ΔV m/sec	13.3	9.9	12.0

TABLE II

RESULTS AT PERIGEE-RMS VALUES

		STANDARD SCHEDULE	DENSE SCHEDULE	OBSERVATION ERROR × 5
	NUMBER OF OBSERVATIONS	77	844	844
MISS	PERIGEE VARIATION	1.1	0.6	1.2
MISS	r(km)	26.4	11.5	33.5
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TABLE I

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UNCERTAINTY	$\tilde{v}$(m/sec)	0.27	0.08	0.32
	TOTAL APPLIED ΔV m/sec	13.3	9.9	12.0

TABLE II

CIRCUMLUNAR MISSION

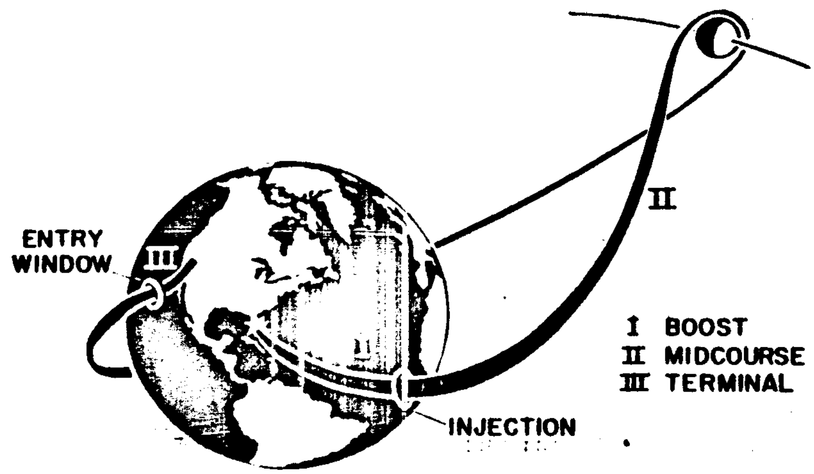

Figure 1

SIMPLIFIED BLOCK DIAGRAM OF GUIDANCE PROBLEM

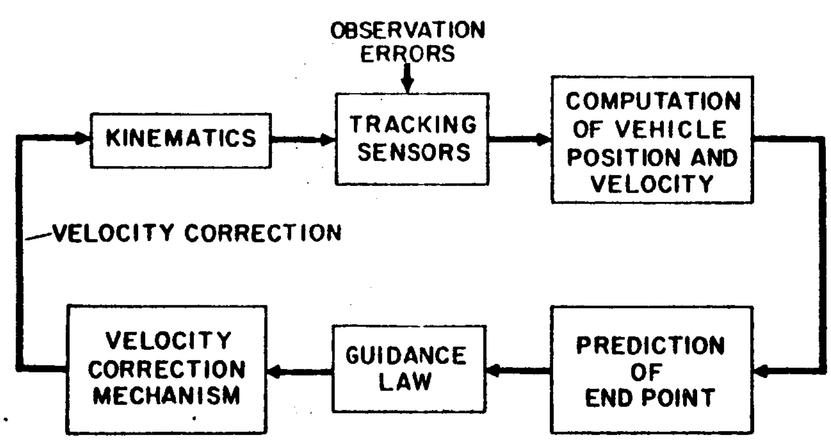

Figure 2.

DETAILED REPRESENTATION OF GUIDANCE SYSTEM

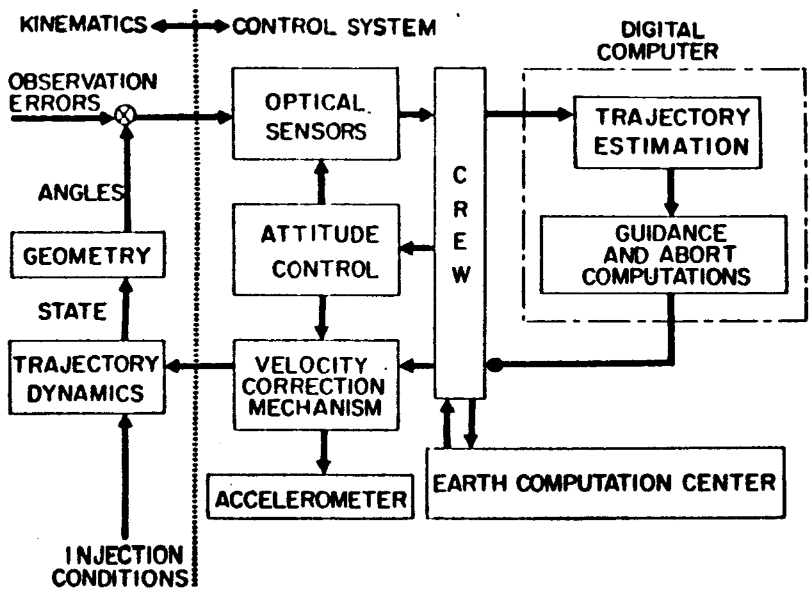

Figure 3.

TRAJECTORY ESTIMATION PROCESS

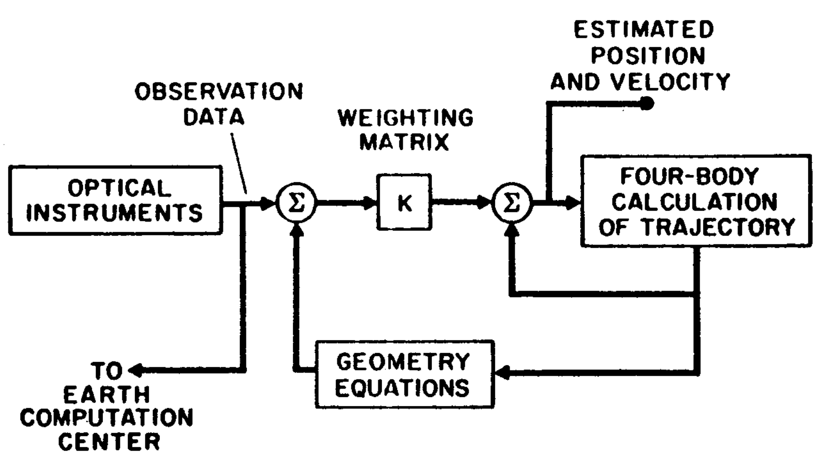

Figure 4.

COMPUTATION OF MIDCOURSE VELOCITY CORRECTIONS

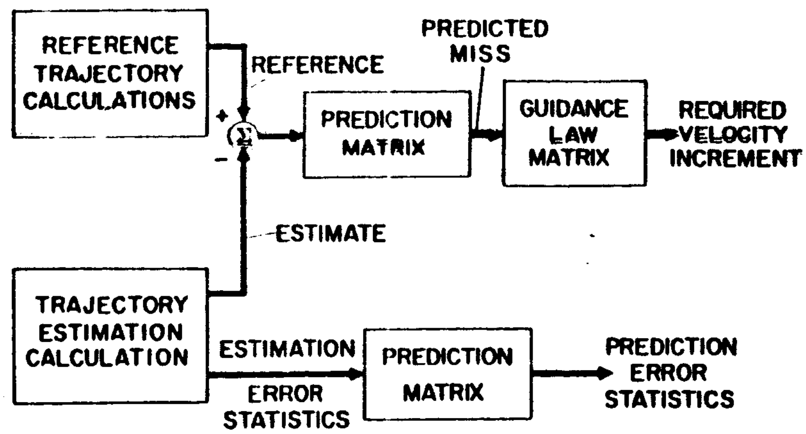

Figure 5.

SCHEDULE OF OBSERVATIONS AND VELOCITY CORRECTIONS

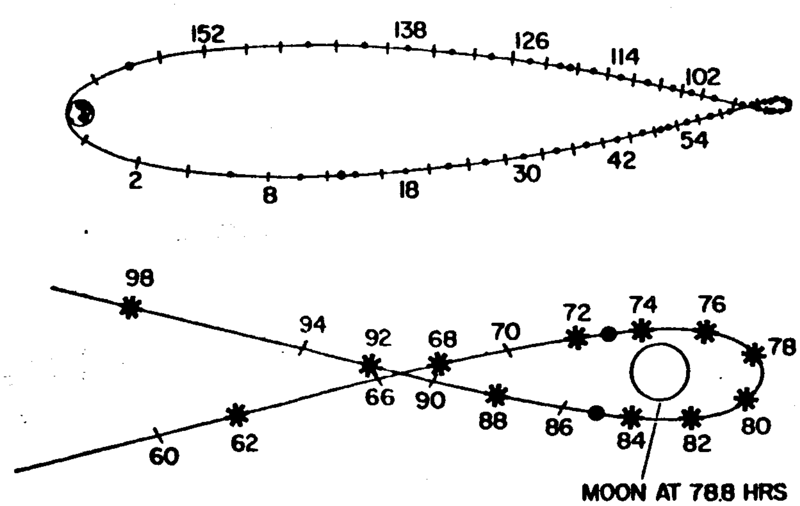

Figure 6

ERRORS AT TIME OF REFERENCE PERIGEE

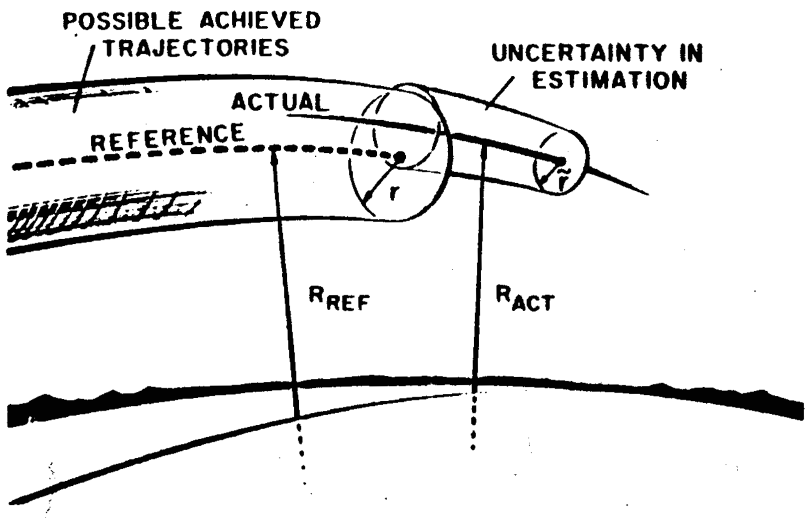

Figure 7.